Triangle Models and Misconceptions

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Abstract

Some misconceptions in the use of the triangle method for remotely estimating surface soil water content and surface evapotranspiration are described. With the correct interpretation of the geometry, simple solutions for surface soil moisture and evapotranspiration fraction can be obtained. In this paper, some possible misinterpretations are pointed out in the application of the triangle method, and it illustrates the use of both simple and complex formulations to obtain solutions is also illustrated.

Keywords

Triangle Method; Estimating Soil Water Content; Estimating Evapotranspiration Fraction

The Triangle

The so-called 'triangle' method for remote sensing of surface properties such as surface soil water content, surface evapotranspiration and surface air temperature, has been widely applied in the United State, China, and elsewhere (Jiang and Islam, 2001; Tang et al., 2010; Shu et al., 2011; Venturi et al., 2004; Piles, 2011, Long and Singh, 2013). It is a powerful and efficient method in that it uses the triangular geometry of the pixel distribution in surface radiant temperature (Tsir) and vegetation fraction (Fr) space to establish boundary conditions for the solution of equations for any given choice of surface energy budget models. Visually or analytically obtained boundary conditions constrain solutions without recourse to detailed knowledge of atmospheric or surface conditions (Carlson, 2007). Moreover, the scaling of measured variables reduces, if not virtually eliminates, the need to correct the measured radiances for atmospheric attenuation (Carlson and Ripley, 1997). Thus, derived parameters are obtained by means of radiative temperature and vegetation index measurements from satellite or aircraft that are largely self contained and internally consistent within the triangular pixel space.

Various methods, both simple and complex, have been published to use remotely sensed measurements to obtain land surface parameters: soil water content and evapotranspiration. Some of these methods discussed in the literature appear to be based on misconceptions or the lack of understanding the triangle method or are unnecessarily complicated. The purpose of this paper is to clarify certain aspects of the triangle method and to suggest some simple mathematical solutions, so as to enhance its practical application.

Triangle from Aircraft Image

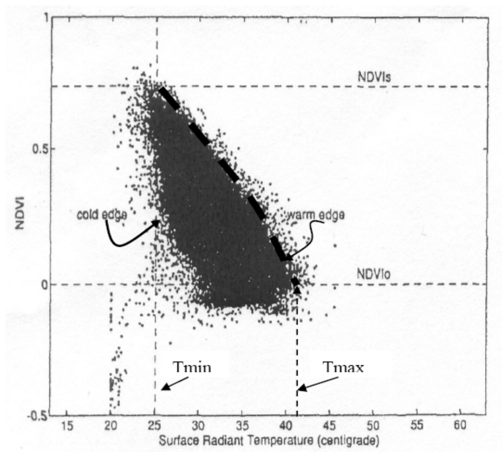

FIG. 1 PIXEL DISTRIBUTION IN NDVI, Tsir SPACE, SHOWING THE VARIOUS LIMITS REFERRED TO IN THE TEXT

Figure 1 shows the observed configuration of the triangle produced by the distribution of remotely measured pixels. Pixels are confined within data-imposed limits: the warm edge, the cold edge, bare soil and dense vegetation. Surface infrared temperature (Tsir) is scaled among the maximum surface radiant temperature, representing dry, bare soil (Tmax), and the minimum temperature representing that of dense vegetation (Tmin). The bare soil and dense vegetation limits are determined for the normalized difference vegetation indices of NDVIo and NDVIs. NDVI can be converted to units of vegetation fraction (Fr) (Carlson, 2007). The warm edge, which can be best defined by visual inspection of the pixel distribution, represents the limit of soil *surface* dryness for a given vegetation amount. It can also be defined analytically. The cold edge represents maximum soil wetness. NDVIs represents the value of fractional vegetation cover

equal to 1.0. Some NDVI points can exceed NDVIs, as in Figure 1, but Fr is constrained to be equal or less than 1.0. NDVIo, the bare soil value, may not necessarily be equal to zero, but it should always correspond to Fr=0.

Misconceptions

It has been noted that some papers may have misinterpreted the physics of the triangle. These possible misconceptions are corrected as follows:

- The warm edge is NOT an isopleth of zero evapotranspiration.
- The warm edge IS a zero isopleth for *soil* evaporation.
- Vegetation, regardless of the soil water content, transpires close to potential, unless it is wilting
- It is not possible with optical measurements to determine deep layer soil water content or the water content of the vegetation or the surface soil water content near the apex of the triangle, near Fr=1.0
- Scaling of temperature and vegetation index is necessary for the following reasons: (1) Fr is a physical variable while NDVI is not; (2) scaling reduces the need to make atmospheric corrections; (3) scaling effectively creates a 'universal' triangle, allowing the shape of the triangle to be maintained from one day to the next, thereby enabling one to follow pixels that migrate with time within the triangle; (4) scaling removes the need to introduce different atmospheric conditions for each day, while minimizing the effects of atmospheric temperature changes from one day to the next.
- Choice of model for transferring pixel measurements to values of surface parameters is not important, but it is easiest and possibly without great loss of accuracy to use the simplest possible algorithm.
- A triangular or trapezoidal shape often emerges when cloud and standing water are removed from the pixel distribution; such removal is not difficult to perform
- One can establish the boundary conditions, Tmax, Tmin, NDVIs and NDVIo even without a large number of pixels, provided that some patches of dry bare soil (as in the center of an urban area) and dense vegetation exist.
- Given these end points, the warm edge can be determined by inspection, or with an algorithm.

Two methods to extract the surface energy fluxes and soil water content are now described. One is a simple geometric solution, the other numerical.

Geometric Solutions

Much effort has been invested in the literature in creating mathematical solutions. Here a purely geometrical solution has been presented, one that is nevertheless based on physical arguments. Consider the schematic triangle in Figure 2. Important variables are the surface soil moisture availability (Mo) and the evapotranspiration fraction (EF= LE/Rn, where Rn is the net radiation at the surface).

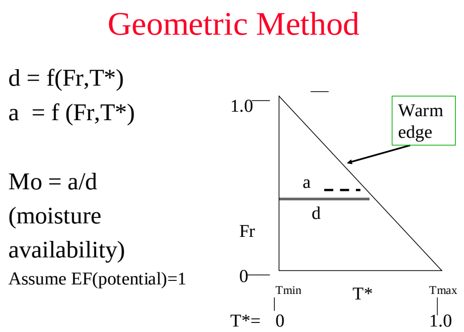

FIG. 2 SIMPLE GEOMETRY OF THE TRIANGLE

They are thusly defined thusly in (1):

$$M_0 = \frac{LE_{soil}}{LE_{soil}(potential)} = \frac{\theta_{soil}}{\theta_{fieldcapacity}}$$

where

$$M_0 = 1 - T^*(pixel) / T^*_{\substack{warm \\ edge}}; \rightarrow M_0 \leq 1.0; \geq 0$$

$$T^*_{\substack{warm \\ edge}} = (1 - Fr)$$

$$EF_{total} = EF_{soil}(1 - Fr) + EF_{veg}(potential) * Fr$$

$$EF_{soil} = Mo$$

$$EF_{veg} = 1.0$$

Here, Mo, the surface moisture availability, is equal to the ratio of the lengths a/d. Both of these lengths are functions of the scaled radiometric surface temperature (T*) and Fr. LE is composed of the transpiration from the vegetation (LEveg), which is taken as the potential transpiration(LEpotential). Evaporation from the soil (LEsoil) varies according to Mo. θ is the actual volumetric soil water content or that at field capacity (θfieldcapacity), T* is the scaled surface radiometric temperature, which is obtained from Tsir scaled between the maximum and minimum temperatures in the triangle and the other variables and subscripts are self evident. T*, Fr, and EF vary from 0 to 1.0.

The geometry is consistent with Jiang and Islam (2001).

Surface soil water content and LE_{soil} are zero at the bare soil and warm edge intersection and remain zero along the warm edge. Soil water content, LE_{soil} and LE_{veg} are at potential along the cold edge over the range of vegetated fraction Fr. Thus, soil moisture availability varies linearly from the cold to the warm edge and the total evapotranspiration (or EF) varies linearly with Fr. The solutions, shown by the isopleths in Figure 3, may or may not be as accurate as those derived from more complex models. They are nevertheless much more easily derived.

SOLUTION FOR EF, Mo

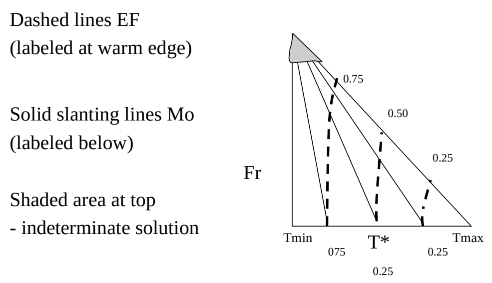

FIG. 3 SIMPLE GEOMETRIC SOLUTION FOR MO (SOLID LINES) AND EF (DASHED LINES). THE SHADED AREA NEAR THE VERTEX SUGGESTS THAT MO IS INDETERMINATE WHERE ITS ISOPLETHS CONVERGE.

A variation of the triangle observed in the pixel data often occurs in the form of a sloping triangle, similar to Figure 1 and depicted in Figure 4. A sloping triangle may mean either that the wet bare soil possesses a higher temperature than the wet vegetation temperature or that no wet soil pixels were found in the image, the latter being the more likely case. Usually, a constant cold edge temperature is chosen in order to simplify the solution

SLOPING TRIANGLE

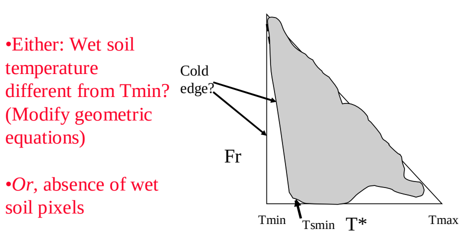

FIG 4 EXAMPLE OF A SLOPING TRIANGLE; SHADING DENOTES PIXEL ENVELOPE

Another variant of the triangle, one with a truncated top, is essentially a trapezoid. This situation may indicate either that no dense vegetation pixels exist within the triangle, in which case Fr is everywhere less than 1.0, or that the temperature of a full vegetation cover varies with soil water content. The latter is more likely, as this type of Mo variation can be modeled at full vegetation cover by simply making soil conductivity (thermal inertia) vary with Mo.

This suggests that the soil is not always quite invisible to the satellite sensor underneath dense vegetation. The important thing to realize here is that the variation of Mo with T* at full vegetation cover does not represent the water content (and therefore the water stress) affecting the vegetation, *i.e.*, the root zone water content.

Numerical Solutions

One may also use a numerical model to simulate the isopleths within the triangle, as reported by Carlson and Sanchez (1999), Carlson (2007), Piles et al. (2011), and many others. Complex solutions derived thusly are more non linear and possibly more accurate that the simple geometric methods but more involved to produce.

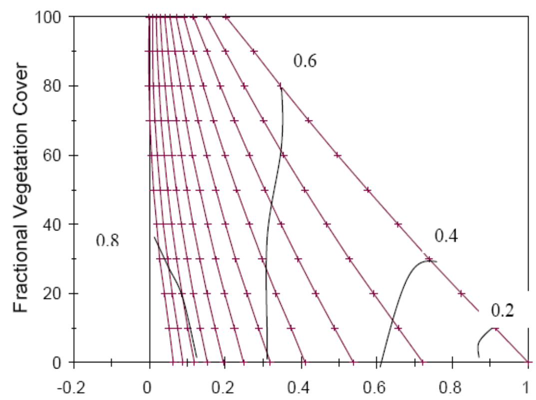

FIG 5 NUMERICAL SOLUTION FOR ISOPLETHS OF Mo (SLOPING SOLID LINES AT INTERVALS OF 0.1, INCREASING FROM THE RIGHT TO LEFT SIDE AND EF (THIN CURVED LINES)

The type of model employed, however, is unimportant, as the boundaries of the triangle tend to impose similar solutions for these variables. Figure 5 shows an example of a numerical solution generated by a soil/vegetation/atmosphere transfer (SVAT) model for a sloping, truncated case from Carlson and Sanchez (1999).

Mo isopleths vary much as in Figure 3, but those for EF differ from the linear variation in Figure 3. This difference in character of the EF isopleths is due primarily to two factors: (1) the numerical simulation allows soil water to migrate to the surface from below (thereby keeping $EF_{soil}>0$ at the point where Fr and Mo are zero), and (2) vegetation does not transpire like a

wet surface because stomatal resistance in the plant always acts to inhibit transpiration to some degree even in well-watered vegetation.

Figure 5 was created in an earlier paper by Carlson and Sanchez-Asofeifa (1999) using a parameter called the *minimum stomatal resistance,* which, for most row crops, is typically between 25 and 50 s m^{-1} and a much larger value for trees. Imposition of the minimum stomatal resistance reduces the potential transpiration from vegetation to something less than that for standing water. This may account for a sloping triangle, but with the opposite tilt from that shown in Figure 4. Carlson and Sanchez-Asofiefa(1999) showed that when the minimum stomatal resistance for trees is used, the isopleths of EF differ noticeably from those for crops but the overall pattern is similar,

Conclusions

The methodology to use the so-called triangle method to estimate surface evapotranspiration and surface soil moisture availability and to elucidate possible misconceptions in its usage has been clarified.

In using either the simplest possible geometric model or a highly complex numerical simulation, the generated isopleths of surface soil moisture availability are very similar, although significant differences exist for evapotranspiration fraction. Whether one method is more accurate than another or whether application of the more complex methodologies is worthy of extra effort given a particular application, are topics to be investigated.

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